Improving the worthiness of the Henry problem as a benchmark for density-dependent groundwater flow models

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[1] This study considers the worthiness of the Henry saltwater intrusion problem as a test case to benchmark density-dependent groundwater flow models. Previously published results from a coupled versus uncoupled analysis indicate that the flow patterns associated with the standard Henry problem are largely dictated by the boundary forcing and not necessarily a result of density-dependent effects [*Simpson and Clement*, 2003]. Results in this study show that the worthiness of Henry's problem can be improved by decreasing the freshwater recharge. This modification maintains the usual benefits of the standard test case, while at the same time increasing the influence of the density-dependent effects. The numerical results for the modified Henry problem are rigorously tested against a new set of semianalytical results. The semianalytical result for the modified problem offers an additional improvement over the standard Henry problem that has typically only been tested using intercode comparisons. *INDEX TERMS:* 1829 Hydrology: Groundwater hydrology; 1832 Hydrology: Groundwater transport; 1831 Hydrology: Groundwater quality; *KEYWORDS:* model development, saltwater intrusion

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1. Introduction

[2] The Henry saltwater intrusion problem concerns a vertical slice through an isotropic, homogeneous, confined aquifer. A constant flux of freshwater is applied to the inland boundary, while the seaward boundary is exposed to a stationary body of higher density seawater. Saltwater intrudes from the sea boundary until an equilibrium between the heavier intruded fluid and the lighter recharging fluid is reached. Figure 1 depicts the domain and boundary conditions. *Henry* [1964] cast the governing equations in a nondimensional form and presented a semianalytical solution method for the steady state distribution of salt concentration and stream function. The Henry problem has been widely used as a test case against which the results from density-dependent groundwater flow models have been benchmarked [e.g., *Frind*, 1982; *Voss and Souza*, 1987; *Simpson and Clement*, 2003].

[3] The Henry problem has enjoyed great popularity as a test case of density-dependent groundwater flow models; however, the problem has been highly controversial [*Segol*, 1994]. This discussion is not intended to recall the entire history of the Henry problem, but a brief discussion of various developments serves to illustrate the significance of the improvements proposed in this work. *Pinder and Cooper* [1970] and *Lee and Cheng* [1974] provided the earliest numerical simulations of Henry's problem. These numerical simulations did not compare well with each other, nor did they compare well with Henry's semianalytical solution. Subsequently, *Segol et al.* [1975] solved the problem using a transient finite element scheme. This work marked the first major change in the Henry problem as it introduced a mixed boundary condition on the seaward boundary to avoid difficulties in the finite element solution near the outflow region of the aquifer. *Segol et al.* [1975] replaced the original homogeneous Dirichlet boundary (Figure 1) with a mixed Dirichlet-Neumann boundary. The upper 20 cm of the new boundary was a zero-concentration flux exit boundary, and the lower 80 cm remained a constant concentration Dirichlet boundary [*Segol et al.*, 1975].

[4] The next major development came from *Voss and Souza* [1987], who claimed that certain discrepancies in the previously published solutions were due, in part, to the use of two different values of the dispersion coefficient. In addition, *Croucher and O'Sullivan* [1995] suggested that truncation error in the previously used numerical grids could have influenced the accuracy of the solutions to the Henry problem. *Croucher and O'Sullivan* [1995] provided the first comprehensive grid convergence study in conjunction with higher accuracy finite difference approximations to conduct an intercode comparison. This study showed that the finite element solutions published by *Frind* [1982] might have been influenced by the presence of numerical dispersion.

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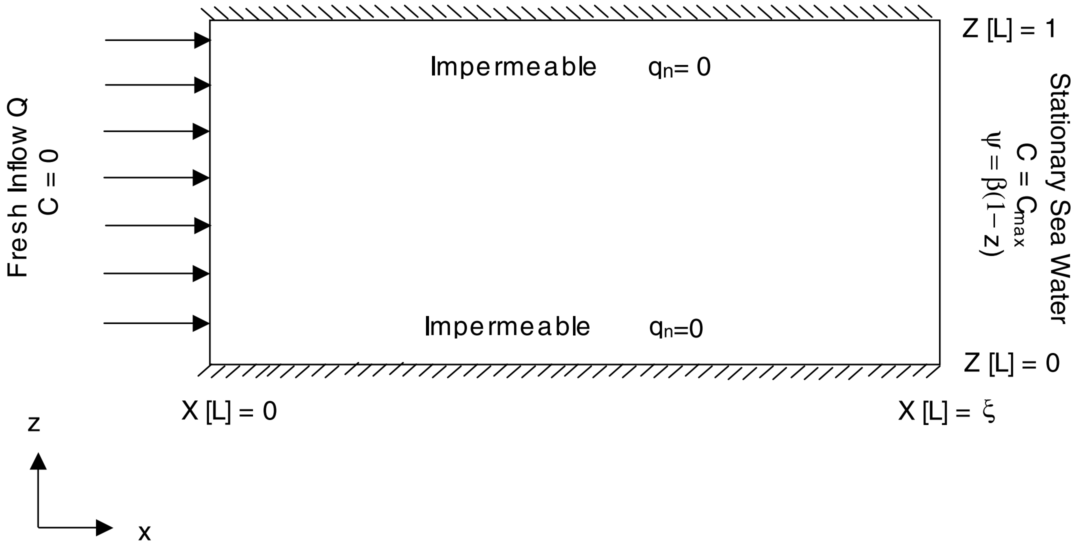

Figure 1. Domain and boundary conditions for the Henry saltwater intrusion problem.

[5] The semianalytical solution developed by Henry was reevaluated and used to show that Henry's original results were in error [*Segol*, 1994]. The error in Henry's analysis was attributed to convergence difficulties in the semianalytical method due of a lack of computing resources. The revised semianalytical solution was compared to a finite element solution which showed that the isochlors in both solutions advanced to the same position along the base of the aquifer [*Segol*, 1994]. Note however, the semianalytical solution used in this comparison utilized the original sea boundary condition while the finite element solution used modified sea boundary conditions. The discrepancy between the two solutions near the exit boundary was expected because of the difference in the boundary conditions [*Segol*, 1994]. Therefore, while this comparison was the first in 30 years to show some agreement between the numerical and semianalytical results, the comparison was flawed because the two solutions were inherently different.

[6] More recently, *Simpson and Clement* [2003] used a coupled versus uncoupled strategy to show that the Henry problem may not be a good choice for testing density-dependent flow models. *Simpson and Clement* [2003] tested the sensitivity of the Henry problem to density-coupled effects by qualitatively comparing density-coupled and density-uncoupled numerical solutions for the problem. The application of this strategy to the Henry problem illustrated that the position and shape of the 50% isochlor was similar whether the density-coupled effects were properly accounted for or not. This suggested that the density-dependent effects shown in the Henry problem are minor in comparison with the influence of the boundary forcing provided by the freshwater recharge. This result is very important, especially when considering that the Henry problem is the one of the most widely used test cases for benchmarking density-dependent flow models. The work of *Simpson and Clement* [2003] suggested the Elder problem of salt convection [*Elder*, 1967] as an alternative to the Henry problem for model testing since the Elder problem was shown to be worthy of the coupled versus uncoupled test. However, it must be acknowledged that the Henry problem will always be used as a test case for density-dependent model development since Henry's problem deals explicitly with saltwater intrusion which remains an important and unresolved issue requiring further research and understanding [*Miller and Gray*, 2002]. Although several other test problems for density-dependent model benchmarking have been devised, none of these problems enable explicit model testing of classical saltwater intrusion processes [e.g., *Elder*, 1967; *Organisation for Economic Co-operation and Development*, 1988; *Simmons et al.*, 1999; *Johannsen et al.*, 2002]. A thorough discussion of the difficulties and shortcomings of these alternative benchmarks have been presented elsewhere [*Diersch and Kolditz*, 2002]. Therefore an investigation into whether the worthiness of the Henry problem can be improved would help alleviate some of these difficulties associated with the Henry problem.

[7] This manuscript presents an improved alternative to the standard Henry saltwater intrusion problem. In recognition of the popularity of the Henry problem along with the associated historical problems, an alternative solution with a reduced freshwater recharge is presented. The reduction of the freshwater recharge means that the density effects play a more significant role in determining the isochlor distribution, this is to be shown by comparing the coupled versus uncoupled simulations for both the standard and modified versions of the Henry problem. Because of problems associated with previous intercode and grid convergence analyses, the numerical results are tested using a reevaluation of Henry's semianalytical method. In addition, certain advantages in using the modified problem are identified in the application of both the numerical and semianalytical solution methods.

2. Governing Equations

[8] The details of the numerical model used in this study are described by *Simpson and Clement* [2003]. A brief account of the governing equations is restated. Density-dependent fluid flow in two-dimensional (vertical) porous

media can be described by the following set of coupled, nonlinear partial differential equations:

$$\frac{\partial(\beta\varphi)}{\partial t} + S_s\beta\frac{\partial\psi}{\partial t} = \frac{\partial}{\partial x}\left(\beta K\frac{\partial\psi}{\partial x}\right) + \frac{\partial}{\partial z}\left(\beta K\frac{\partial\psi}{\partial z}\right) + \frac{\partial(\beta^2 K)}{\partial z} \quad (1)$$

$$\varphi\frac{\partial C}{\partial t} = \beta\frac{\partial}{\partial x}\left(D_x\varphi\frac{\partial C}{\partial x}\right) + \beta\frac{\partial}{\partial z}\left(D_z\varphi\frac{\partial C}{\partial z}\right) - V_x\varphi\frac{\partial C}{\partial x} - V_z\varphi\frac{\partial C}{\partial z} \quad (2)$$

Where φ is the porosity of the porous medium, β is the ratio of the fluid density to a reference freshwater density, K [LT^{-1}] is the hydraulic conductivity of the porous medium, ψ [L] is the freshwater pressure head of the fluid, S_s [L^{-1}] is the specific storage coefficient for the porous medium, C [ML^{-3}] is the concentration of the solute which contributes to the density variation, D_i [L^2T^{-1}] is the total dispersion coefficient in the ith Cartesian direction and V_i [LT^{-1}] is the fluid velocity in the ith Cartesian direction. The fluid density is assumed to vary as a linear function of the solute concentration. Further details of the numerical model including a rigorous description of the Galerkin finite element numerical solution procedure is given by *Simpson and Clement* [2003].

3. Semianalytical Method

[9] For the semianalytical method, *Henry* [1964] presented a governing equation for steady state fluid flow and a steady state advection-dispersion equation for the continuity of dissolved salt. The density variation was assumed to be a linear function of the dissolved salt concentration. *Henry* [1964] simplified the governing equations by assuming that the dispersive processes in the aquifer could be represented by a constant dispersion coefficient. In addition, Henry invoked the Boussinesq approximation for the flow equation by noting that the maximum fluid density was nowhere significantly greater than the freshwater density. This approximation implied the existence of a stream function for the fluid flow [*Henry*, 1964].

[10] *Henry* [1964] showed that the three significant dimensionless parameters for the problem were the aspect ratio of the domain $\xi = \frac{w}{d}$, $a = \frac{Q}{k_1 d}$ and $b = \frac{D}{Q}$. Note that Q [L^2T^{-1}] is the freshwater recharge per unit width of coast, w [L] is the horizontal length of the domain and d [L] is the depth of the domain. $k_1 = K(\frac{\rho_s - \rho_0}{\rho_0})$ where K [LT^{-1}] is the saturated hydraulic conductivity, ρ_0 [ML^{-3}] is the freshwater density, ρ_s [ML^{-3}] is the saltwater density and D [L^2T^{-1}] is the coefficient of dispersion. Henry applied the following homogeneous boundary conditions to the governing equations (Figure 1):

$$\begin{aligned} \frac{\partial\bar{C}}{\partial z} &= 0 \quad \text{at } z = 0, 1 \\ \bar{\Psi} &= 0 \quad \text{at } z = 0, \\ \bar{\Psi} &= 1 \quad \text{at } z = 1 \\ \frac{\partial\bar{\Psi}}{\partial x} &= 0 \quad \text{at } x = 0, \xi \\ \bar{C} &= 0 \quad \text{at } x = 0, \\ \bar{C} &= 1 \quad \text{at } x = \xi \end{aligned} \quad (3)$$

where $\bar{C}$ is the nondimensional concentration and $\bar{\Psi}$ is the nondimensional stream function. *Henry* [1964] introduced a change of variables and assumed that solution of the governing equations took the form of a double Fourier series:

$$\begin{aligned} \bar{\Psi} &= \sum_{m=1}^{\infty}\sum_{n=0}^{\infty} A_{m,n}\sin(m\pi z)\cos\left(n\pi\frac{x}{\xi}\right) \\ \bar{C} &= \sum_{r=0}^{\infty}\sum_{s=1}^{\infty} B_{r,s}\cos(r\pi z)\sin\left(s\pi\frac{x}{\xi}\right) \end{aligned} \quad (4)$$

The Fourier coefficients were evaluated using Galerkin's method (Note that this is distinct from the Galerkin finite element method used to obtain the numerical solution in this work). Galerkin's method required that the Fourier expressions (4) were substituted into the governing equations and then multiplied by certain problem specific functions [*Henry*, 1964]. The resulting equations were then integrated across the domain to incorporate the boundary conditions (3). This procedure yields an infinite set of nonlinear algebraic equations for the Fourier coefficients [*Segol*, 1994].

[11] The solution of the system for the Fourier coefficients requires two steps. First the infinite system must be truncated, and second the resulting nonlinear system of equations must be solved. The truncation of the infinite terms is an important step as beyond this point the solution scheme is no longer a true analytical method. Therefore the solution method is more correctly referred to as a semianalytical method. *Segol* [1994] investigated the influence of truncation and proposed a scheme using 183-terms that expanded the original truncation used by *Henry* [1964]. A 203-term truncation scheme similar to that suggested by *Segol* [1994] was used for this work:

$A_{1,0}$ through $A_{1,20}$

$A_{2,1}$ through $A_{2,20}$

$A_{3,0}$ through $A_{3,20}$

$A_{4,1}$ through $A_{4,20}$

$A_{5,0}$ through $A_{5,20}$

$B_{0,1}$ through $B_{0,20}$

$B_{1,1}$ through $B_{1,20}$

$B_{2,1}$ through $B_{2,20}$

$B_{3,1}$ through $B_{3,20}$

$B_{4,1}$ through $B_{4,20}$

Following truncation, the nonlinear system was approximated as a linear system in terms of the $A_{g,h}$ and $B_{g,h}$ coefficients. The quadratic term involving the quadruple sum was treated as a known quantity and an iteration scheme was used to update the coefficients [*Segol*, 1994]. The linear systems generated in this iterative scheme were solved using LU factorization with pivoting.

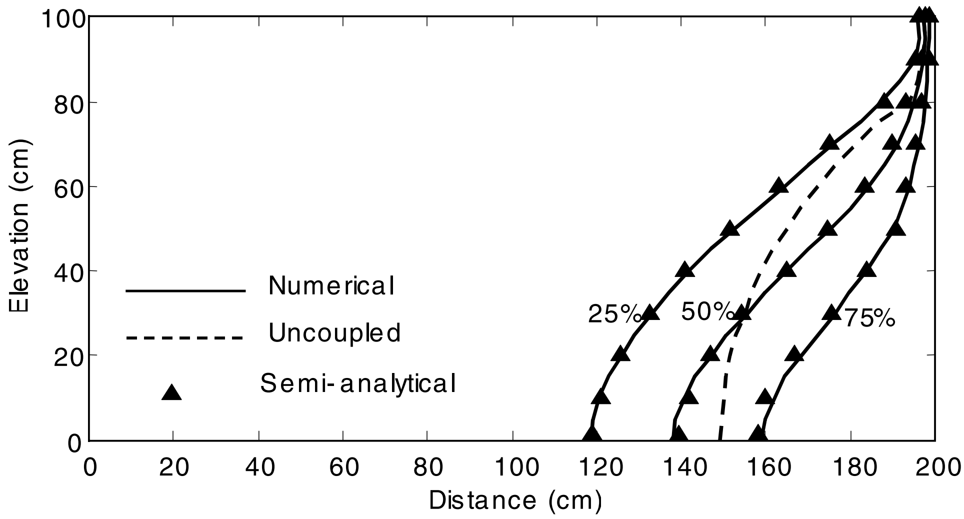

Figure 2. Comparison of numerical and semianalytical results for the standard Henry saltwater intrusion problem a = 0.263, b = 0.1, and ξ = 2.0. The uncoupled 50% numerical isochlor (dashed) is also shown.

[12] The iterative solution technique described by *Henry* [1964] and *Segol* [1994] was adopted. The scheme involved several subiterations where the $B_{g,h}$ $g \neq 0$ terms and $B_{0,h}$ terms were repeatedly evaluated until the sum of the changes in the absolute value of the $B_{g,h}$ terms between successive subiterations fell below some tolerance. The outer-iterative cycle consisted of reevaluating the $A_{g,h}$ coefficients each time the set of $B_{g,h}$ coefficients was considered to have converged. The outer-iterative cycle was considered to have converged when the sum of the changes in the absolute value of the $A_{g,h}$ terms between successive outer-iterations fell below some tolerance.

[13] *Henry* [1964] and *Segol* [1994] noted that the rate of convergence of the solution scheme was dependent upon the value of the nondimensional parameters a and b. The solution where a = 0.263 and b = 0.2 can be obtained very quickly using an initial guess where all the coefficients are zero [*Segol*, 1994]. Once the solution for a = 0.263 and b = 0.2 has been established, this serves as an initial guess from which solutions for other parameterizations can be obtained by considering small stepwise changes in either the value of a or b and then applying the iteration scheme previously described.

4. Coupled Versus Uncoupled Methodology

[14] The work of *Simpson and Clement* [2003] described a coupled versus uncoupled strategy to assess the rigor of test cases used to benchmark numerical models of density-dependent groundwater flow. The coupled versus uncoupled test is undertaken by performing a full density-coupled simulation using the numerical model (equations (1) and (2)) and then comparing the results where the coupling between the groundwater flow equation (equation (1)) and the solute transport equation (equation (2)) is purposely ignored. The uncoupling is achieved by setting the value of β = 1 in the system (1)–(2). Under the uncoupled conditions the solute transport no longer influences the flow equation as the solute acts as a passive tracer.

[15] *Simpson and Clement* [2003] showed that the comparison of the coupled versus uncoupled profiles was a powerful tool for assessing the importance of density coupling for the *Henry* [1964] and *Elder* [1967] problems. The idea of the analysis is that in situations where the coupled and uncoupled profiles are similar, density-coupled processes are not the key feature that determines the distribution of solute concentration and fluid pressure. Alternatively, where the coupled and uncoupled profiles are distinct, the density-coupled processes are critical in determining the solute concentration and pressure distributions. The coupled versus uncoupled analysis is used in this work as a quantitative design tool to improve the worthiness of the Henry problem by proposing a modified case where the difference between the coupled and uncoupled solutions is increased.

5. Results

5.1. Standard Henry Saltwater Intrusion Problem

[16] The semianalytical results for the standard Henry problem correspond to those originally analyzed by *Henry* [1964]. Henry chose the dimensionless parameters ξ = 2.0, a = 0.263 and b = 0.1 as a compromise between those parameters which corresponded to field applications and those which were conducive to obtaining a convergent iterative solution of the nonlinear equations. The semianalytical method was evaluated successfully using 10 outer-iterations with 4 to 5 subiterations each. The Fourier expression for the steady state dimensionless concentration distribution was then computed and interpolated upon a uniform finite element grid composed of 41 $\times$ 21 nodes. The grid was made up of uniformly aligned linear triangular elements with the diagonal pointing in the NE-SW direction. The concentration distribution was obtained by interpolation using standard linear triangular finite element shape functions [*Simpson and Clement*, 2003]. The steady state position of the 25%, 50% and 75% isochlors are shown in Figure 2.

[17] The numerical simulation of the standard Henry saltwater intrusion problem was undertaken by solving the system (1)–(2). Table 1 summarizes the parameters used in the numerical simulation. The solution was found upon a 200 cm $\times$ 100 cm domain. The domain was discretized

Table 1. Aquifer Properties Used for the Numerical Simulation of the Standard Henry Saltwater Intrusion Problem $a = 0.263$, $b = 0.2$, and $\xi = 2.0$

Symbol	Quantity	Value	Unit
D_m	coefficient of molecular diffusion	1.886×10^{-5}	$m^2\ s^{-1}$
α_L	longitudinal dispersivity	0.0	m
α_T	transverse dispersivity	0.0	m
g	gravitational acceleration	9.80	$m\ s^{-2}$
K	hydraulic conductivity	1.0×10^{-2}	$m\ s^{-1}$
Q	recharge per unit width	6.6×10^{-5}	$m^2\ s^{-1}$
S_s	specific storage	0.0	m^{-1}
β_{max}	maximum density ratio	1.025	
φ	porosity	0.35	
ρ_o	reference density	1000	$kg\ m^{-3}$
ρ_{max}	saltwater density	1025	$kg\ m^{-3}$

using 861 nodes and 1600 uniformly aligned linear triangular elements with the diagonal pointing in the NE-SW direction. This corresponds to a uniform discretization with $\Delta x = \Delta z = 0.05$ m. No flow conditions were specified on the upper and lower boundaries. The seaward boundary was held at a hydrostatic pressure distribution and the upstream boundary was subject to the freshwater recharge. In terms of solute transport, the upper and lower boundaries were represented as zero-concentration flux boundaries, while the upstream and downstream boundaries were simulated using $C = 0$ and $C = C_{max}$ Dirichlet boundaries respectively.

[18] A transient simulation was performed with the initial conditions comprising a hydrostatic aquifer containing only freshwater. Temporal integration of the discretized finite element equations was achieved with a constant time step of 12 seconds and a fully implicit time weighting scheme. The iterative coupling of the flow and transport components of the system (1)–(2) was considered complete when the maximum change in the freshwater pressure head fell below $\Delta\psi = 5 \times 10^{-5}$ m between successive iterations [*Simpson and Clement*, 2003]. The system was assumed to come to a steady state after approximately 160 minutes of simulation when the position of the toe of the 25%, 50% and 75% isochlors had become stationary (Figure 2). Comparing the position of the isochlors reveals a good correspondence between the numerical and semianalytical results, both the shape and position of the isochlors compare very well.

[19] It is interesting to note that the comparison of the numerical and semianalytical results in Figure 2 is similar to that presented by *Segol* [1994]. However, this is the first time that a complete correspondence between the semianalytical and numerical solutions has been shown. In the comparison presented by *Segol* [1994] the numerical solution was obtained using a modified boundary condition and therefore the solutions did not correspond near the outflow portion of the sea boundary. One of the major limitations of this previous comparison was that the profiles could only be compared in the lower 80 cm of the domain due to the influence of the different boundary conditions [*Segol*, 1994]. This limitation was not encountered in the present work, which has shown an excellent agreement between the numerical and semianalytical distribution of the steady state isochlors over the total aquifer depth.

[20] One further difficulty encountered in the numerical simulation of Henry's problem is in the representation of the solute transport processes in the top right hand outflow region of the aquifer. This area is dominated by convergent outward flowing fluid and so represents a region where advective transport of the solute can dominate the solute transport processes. Therefore previous investigators have noticed that it is possible to experience oscillations in the numerical solution of the advection-dispersion equation in this region when using standard numerical approximations to the governing equations [*Segol et al.*, 1975; *Croucher and O'Sullivan*, 1995]. *Segol et al.* [1975] attempted to overcome this problem by introducing a mixed boundary condition. However, this alternative is not suited when using the semianalytical solution to test the numerical results since the semianalytical solution requires the use of homogeneous boundary conditions. The issue of numerical errors in the outflow region will be further discussed in section 5.3.

5.2. Coupled Versus Uncoupled Solution for the Standard Henry Saltwater Intrusion Problem

[21] Once the density-coupled solution to the standard Henry problem has been obtained, it is possible to compare the coupled solution to the uncoupled solution. The uncoupled solution is obtained by solving the problem numerically, but ignoring the coupling between the groundwater flow and solute transport equations [*Simpson and Clement*, 2003]. The uncoupled solution was obtained using the same numerical discretizations used for the coupled solution; the uncoupled profile was steady after 160 minutes of simulation. Figure 2 compares the 50% isochlors for the coupled and uncoupled profiles. The comparison shows that in the uncoupled solution the solute intrudes into the aquifer and takes on a shape and position that is similar to the true coupled solution in a qualitative sense. Note that *Simpson and Clement* [2003] observed similar results, however that previous work was concerned with the version of Henry's problem involving *Segol et al.*'s [1975] mixed seawater boundary condition and a reduced dispersion coefficient so that the model results could be compared to those of *Frind* [1982].

[22] Because of the similarity of the coupled and uncoupled solutions to the standard Henry problem, it seems plausible that a faulty algorithm could produce results that look similar to the true solution when in fact the results might be erroneous [*Simpson and Clement*, 2003]. This complication has been further compounded by the publication of several sets of significantly different results in the literature, all of which are broadly grouped as solutions to Henry's problem. In addition, the similarity between the coupled and uncoupled solutions to the Henry problem means that density-coupled effects are not a major control on the solution of the problem. Therefore the standard Henry problem is a poor test case for demonstrating the ability of a variable density groundwater model to simulate density-coupled processes. It is therefore desirable to formulate a modified version of the Henry problem that can alleviate the similarity between the coupled and uncoupled solutions.

5.3. Modified Henry Saltwater Intrusion Problem

[23] In an attempt to improve the worthiness of the Henry problem, an alternative approach was taken where the recharge rate of the freshwater was halved. Since the objective of modifying the problem was to increase

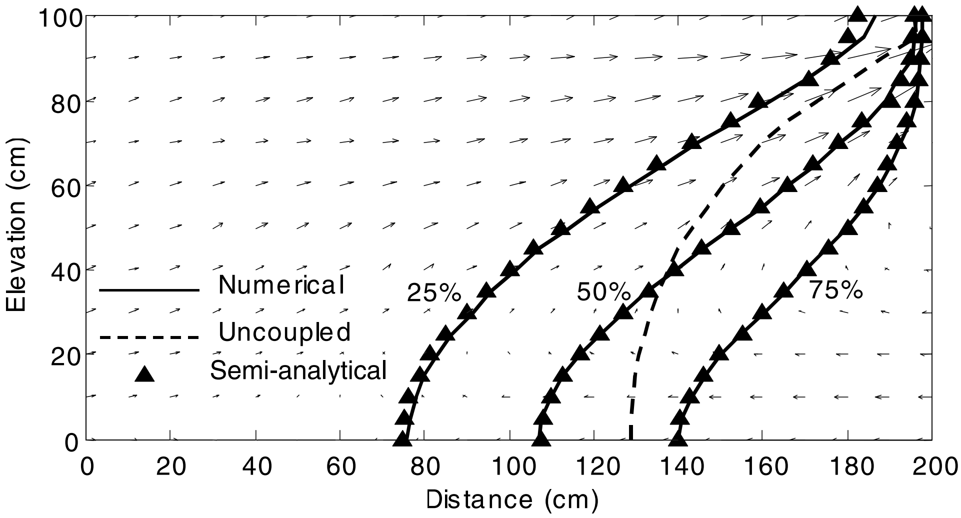

Figure 3. Comparison of numerical and semianalytical results for the modified Henry saltwater intrusion problem a = 0.1315, b = 0.2, and ξ = 2.0. The velocity distribution and uncoupled 50% numerical isochlor (dashed) are also shown. See color version of this figure at back of this issue.

the relative importance of the density-dependent effects as compared to the boundary forcing, there appears to be two possible methods available to achieve this. Firstly, it would be possible to increase the density of the invading fluid. While this option would increase the relative importance of the density effects and would not prevent the use of a numerical simulation, the application of Henry's semianalytical method relies upon invoking the Boussinesq approximation [*Henry*, 1964]. This approximation is only valid for flows where the density variation is small, and therefore the semianalytical result is not applicable when the density of the invading fluid is increased. Secondly, the alternative option for increasing the influence of the density effects is to decrease the boundary forcing. This second option was chosen since it would not invalidate the use of Henry's semianalytical solution.

[24] A trial-and-error approach was used to determine by how much the forcing should be reduced. A reduction of the recharge rate by a factor of two was chosen since this yields a steady solution whereby the 50% isochlor advances to almost halfway along the floor of the aquifer. Note that it would be possible to reduce the recharge rate by more than a factor of two; however, under these conditions the isochlors would advance further into the aquifer and would eventually be influenced by the zero-concentration Dirichlet boundary condition specified on the upstream boundary (Figure 1). Although it would be possible to resolve the problem on a domain with a wider aspect ratio to avoid this complication, it was considered desirable to maintain an aspect ratio of ξ = 2.0 in the design of the modified problem. The advantage of maintaining the same aspect ratio is that both the modified and standard version of Henry's problem can then be solved on the same domain without the need to construct a second mesh.

[25] The semianalytical solution for the modified Henry's problem was obtained using the dimensionless parameters ξ = 2.0, a = 0.1315 and b = 0.2. Note that the new solution was obtained by using the solution for a = 0.263 and b = 0.2 as an initial distribution of the Fourier coefficients, and then lowering the value of a stepwise until the desired solution was obtained. The nondimensional parameter a was lowered using 10 nonuniform steps: each stepwise reduction in a required up to 9 outer-iterations in order to achieve convergence. Each of the outer-iterations in turn required up to 9 subiterations to achieve convergence. Generally, convergence became slower as the value of a was reduced. In order to overcome this, smaller stepwise changes were taken as the value of a decreased. Once the converged solution was obtained, the concentration was evaluated and interpolated onto the finite element grid (Figure 3). A broader distribution of the nondimensional concentration is presented in Table 2 for benchmarking purposes. For brevity, only a sample of the semianalytical results in the higher concentration region of the aquifer is presented in Table 2. The tabulated distribution of the nondimensional concentration is sufficient to test numerical results for the concentration in the region $25\% \leq \bar{C} \leq 100\%$.

[26] It is of interest to consider the influence of reducing the nondimensional parameter a upon the convergence of the semianalytical iterative scheme. Convergence was much slower when we halved the a parameter from a = 0.263 and b = 0.2 to a = 0.1315 and b = 0.2 (modified conditions) than when we halved the b parameter from a = 0.263 and b = 0.2 to a = 0.263 and b = 0.1 (standard conditions). This indicates that Henry's iterative scheme is more sensitive to reductions in a than b. These observations extend those reported by *Segol* [1994] who noted that convergence slowed as the value of b decreased. The similarity of the influence of a and b upon the iterative scheme is not surprising since both parameters play similar roles in the system of nonlinear equations generated by the application of Henry's semianalytical solution method [*Segol*, 1994]. This observation is important if the semianalytical solution is to be used where the nondimensional parameters a and/or b are further decreased. The present analysis has shown that the use of a 203-term truncation is sufficient; however, it seems likely that any future analysis for further reduced values of a and/or b shall require more terms in the Fourier series to be retained for the semianalytical iterative scheme to converge. As *Henry* [1964] noted, it is not possible in

Table 2. Distribution of the Semianalytical Dimensionless Concentration for the Modified Henry Saltwater Intrusion Problem $a = 0.1315$, $b = 0.2$, and $\xi = 2.0$

	x = 75	x = 85	x = 95	x = 105	x = 115	x = 125	x = 135	x = 145	x = 155	x = 165	x = 175	x = 185	x = 195
z = 100 cm	0.01	0.05	0.03	0.07	0.06	0.10	0.09	0.14	0.14	0.19	0.20	0.27	0.43
z = 90 cm	0.02	0.05	0.04	0.07	0.07	0.11	0.11	0.15	0.16	0.22	0.24	0.32	0.51
z = 80 cm	0.03	0.06	0.06	0.09	0.10	0.13	0.15	0.19	0.22	0.28	0.34	0.45	0.69
z = 70 cm	0.05	0.07	0.09	0.12	0.14	0.18	0.21	0.26	0.31	0.38	0.47	0.60	0.84
z = 60 cm	0.08	0.10	0.13	0.16	0.20	0.24	0.29	0.35	0.42	0.50	0.59	0.72	0.91
z = 50 cm	0.10	0.14	0.18	0.22	0.27	0.32	0.38	0.45	0.52	0.61	0.70	0.81	0.92
z = 40 cm	0.14	0.18	0.23	0.29	0.34	0.41	0.47	0.55	0.62	0.71	0.79	0.87	0.95
z = 30 cm	0.18	0.23	0.29	0.36	0.42	0.49	0.57	0.64	0.71	0.79	0.86	0.92	0.98
z = 20 cm	0.22	0.28	0.35	0.42	0.50	0.57	0.65	0.71	0.78	0.84	0.90	0.95	0.99
z = 10 cm	0.25	0.32	0.40	0.48	0.55	0.63	0.70	0.77	0.82	0.88	0.93	0.96	0.99
z = 0 cm	0.26	0.34	0.41	0.49	0.57	0.65	0.72	0.78	0.84	0.89	0.93	0.96	0.98

advance to determine how many terms in the scheme need to be retained to obtain a convergent solution, and so, an investigation of the convergence behavior of the iterative scheme for alternative parameterizations remains an open topic for future analysis.

[27] The numerical simulation of the modified Henry problem was performed using the same spatial and temporal discretization used for the standard solution. Convergence of the iterative solution of the flow and solute transport equations was determined using the same criteria. The only change was that the imposed freshwater recharge was reduced to $Q = 3.3 \times 10^{-5}$ m^2 s^{-1}. The transient simulation was conducted using a fully implicit time weighting scheme, the simulation was performed for 280 minutes after which the position of the 25%, 50% and 75% isochlors had become steady (Figure 3). Comparing the numerical and semianalytical profiles reveals an excellent correspondence between the two solution techniques.

[28] Several important advantages can be identified in using the modified Henry problem as compared to the standard solution in terms of both the numerical and semi-analytical methods. Under modified conditions, comparing the numerical results with the semianalytical results near the top of the aquifer is made easier because the isochlor distribution is more diffuse. For example, in the standard Henry problem the 25%, 50% and 75% isochlors are all constricted toward the outflow region of the aquifer (Figure 2). For the modified results, the isochlors are more spread out in this region. Since the isochlors are more widely distributed, it is easier to identify the relative accuracy of numerical results in this region.

[29] In terms of the numerical solution, the modified Henry problem is favorable since the reduced recharge can help alleviate problems with the oscillation in the solution of the advection-dispersion equation near the outflow boundary. Because the modified version of the Henry problem involves a reduced fluid inflow, the amount of fluid being removed from the aquifer at the top of the sea boundary must also be reduced under steady state conditions. The reduction in fluid outflow means that the velocity and advective transport of solute is also reduced near the outflow boundary.

[30] In terms of the numerical results, the horizontal outflow velocity at the top right-hand corner node is reduced from 1.55×10^{-3} m s^{-1} for the standard Henry solution, to 1.06×10^{-3} m s^{-1} under modified conditions. Note that the maximum total velocity ($V_{total} = \sqrt{V_x^2 + V_z^2}$) occurs at this node for both the modified and standard Henry problems. Since these simulations were performed on identical grids with a uniform dispersion coefficient, this 32% reduction in the maximum outflow velocity corresponds to a 32% reduction in the maximum grid Peclet number, $Pe = \frac{V_x \Delta x}{D}$. Note that the total velocity along the sea boundary of the Henry problem is only composed of a horizontal flow component because of the hydrostatic boundary condition. The maximum grid Peclet number is reduced from Pe = 4.1 under the standard conditions to Pe = 2.8 for the modified conditions on this 41 × 21 grid. This decrease in the relative importance of advective transport near the outflow boundary means that standard numerical schemes used to approximate the advection-dispersion equation are less likely to suffer from numerical errors in this region under the modified conditions. This observation is consistent with that of *Voss and Souza* [1987], who claimed that the grid Peclet number should be less than four to avoid oscillatory solutions.

5.4. Coupled Versus Uncoupled Solution for the Modified Henry Saltwater Intrusion Problem

[31] To obtain the uncoupled solution to the modified Henry problem, the numerical algorithm was executed using the same numerical discretizations as for the coupled solution; however, the coupling between the flow and solute transport components of the model was ignored. The algorithm was executed for the same length of time, by which the uncoupled profile had become steady (Figure 3). The uncoupled isochlor intrudes into the aquifer as for the standard uncoupled solution; however, the modified profile shows an exaggerated difference between the coupled and uncoupled solutions. Note in Figures 2 and 3 that the difference between the coupled and uncoupled solutions can be quantified by measuring the horizontal distance between the corresponding coupled and uncoupled isochlors, this distance is denoted δ in this study.

[32] The increased difference in the coupled versus uncoupled profiles for the modified Henry problem can be observed qualitatively. For example, the coupled isochlor has a more pronounced and sharpened toe than the uncoupled isochlor in the modified case. The increase in discrepancy between the coupled and uncoupled solutions for the modified problem can also be demonstrated quantitatively. In the past, a common point of comparison used in intercode studies of the Henry problem has been to compare the position where the 50% isochlor intersects the base of

Table 3. Comparison of the Location Where the 25%, 50%, and 75% Isochlors Intersect the Base of the Aquifer for the Standard and Modified Henry Saltwater Intrusion Problem

	Standard			Modified		
	Coupled	Uncoupled	Difference (δ)	Coupled	Uncoupled	Difference (δ)
25%	x = 118.6 cm	x = 136.7 cm	18.1 cm	x = 75.8 cm	x = 110.7 cm	34.9 cm
50%	x = 138.0 cm	x = 149.4 cm	11.4 cm	x = 107.3 cm	x = 128.7 cm	21.4 cm
75%	x = 159.0 cm	x = 160.5 cm	1.5 cm	x = 140.0 cm	x = 143.7 cm	3.7 cm

the aquifer [*Segol*, 1994]. In comparing the coupled versus uncoupled solution for the standard Henry problem, the coupled solution intersects the base at x = 138.0 cm, while the uncoupled solution intersects at x = 149.4 cm, giving a discrepancy of δ = 11.4 cm. For the modified solution, the position of the intersection of the 50% isochlor under coupled conditions is at x = 107.3 cm, while the uncoupled solution intersects at x = 128.7 cm, thereby giving an increased discrepancy of δ = 21.4 cm. The increase in the discrepancy between the coupled and uncoupled solutions indicates that the modified Henry problem has an improved worthiness when compared to the standard Henry problem since the relative importance of the density-dependent effects have been increased. Table 3 compares the position of the coupled and uncoupled 25%, 50% and 75% isochlors along the base of the aquifer for both the standard and modified versions of the Henry saltwater intrusion problem. The results consistently show that the deviation between the coupled and uncoupled profiles for the modified version of the Henry problem is always higher than for the standard case.

[33] In addition to comparing the position of the isochlors at the base of the aquifer, the location of the 25%, 50% and 75% isochlors were then compared over the entire aquifer depth for both the standard and modified Henry problems. Results are shown in Tables 4–6 where the interpolated position of the 25%, 50% and 75% isochlors are given at each vertical 10 cm interval throughout the domain. The difference is reported as the absolute value of the difference between the x coordinates of the isochlors under coupled and uncoupled conditions. The difference is then summed across the entire profile to give an overall measure of the discrepancy. As before, this analysis indicates that the deviation between the coupled and uncoupled isochlor profiles is greater for the modified problem on average throughout the domain. Note that this study only quantifies the difference in the coupled and uncoupled solutions in a horizontal sense and it should be acknowledged that it would be possible to obtain similar results by measuring the differences in a vertical sense; however, for brevity only the horizontal data are presented.

[34] Further subtleties in maximizing the rigor of the model testing procedure using the modified Henry problem can be inferred by comparing the data in Tables 4–6. The maximum difference between the coupled and uncoupled solutions was always associated with the 25% isochlor. Conversely, the minimum difference between the coupled and uncoupled solutions was consistently associated with the 75% isochlor. This indicates that the relative importance of density-driven features in the Henry problem are spatially variable within the plume and so the rigor of the model testing procedure can be increased by focusing upon a models ability to replicate the isochlors in the low-concentration region of the plume. This observation is important, since for the first time it enables analysts to physically justify how numerical data can be selected to optimize the rigor of the model testing procedure.

5.5. Physical Interpretation

[35] In addition to analyzing the numerical data to compare the differences between the coupled and uncoupled solutions to the Henry problem, it is also possible to physically relate the results to Henry's original nondimensional parameter *a*:

$$a = \frac{Q}{dK\left(\frac{\rho_s - \rho_0}{\rho_0}\right)} \tag{5}$$

Henry's parameter *a*, can be thought of as a dimensionless number that reflects a balance of the advective flux in the

Table 4. Comparison of the Difference Between the Position of the Coupled and Uncoupled 25% Isochlors for Both the Modified and Standard Henry Saltwater Intrusion Problem

	Standard			Modified		
	Coupled	Uncoupled	Difference (δ)	Coupled	Uncoupled	Difference (δ)
z = 0 cm	x = 118.6 cm	x = 136.7 cm	18.1	x = 75.8 cm	x = 110.7 cm	34.9
z = 10 cm	x = 120.7 cm	x = 137.1 cm	16.4	x = 77.9 cm	x = 111.0 cm	33.1
z = 20 cm	x = 125.6 cm	x = 138.8 cm	13.2	x = 83.3 cm	x = 112.5 cm	29.2
z = 30 cm	x = 132.9 cm	x = 141.6 cm	8.7	x = 91.1 cm	x = 115.0 cm	23.9
z = 40 cm	x = 142.1 cm	x = 145.1 cm	3.0	x = 101.4 cm	x = 118.6 cm	17.2
z = 50 cm	x = 152.6 cm	x = 150.3 cm	2.3	x = 114.1 cm	x = 123.5 cm	9.4
z = 60 cm	x = 164.4 cm	x = 157.4 cm	7.0	x = 128.7 cm	x = 130.0 cm	1.3
z = 70 cm	x = 176.4 cm	x = 165.8 cm	10.6	x = 145.0 cm	x = 137.8 cm	7.2
z = 80 cm	x = 187.4 cm	x = 177.6 cm	9.8	x = 161.6 cm	x = 148.4 cm	13.2
z = 90 cm	x = 195.7 cm	x = 195.2 cm	0.5	x = 177.1 cm	x = 160.9 cm	16.2
z = 100 cm	x = 196.1 cm	x = 196.1 cm	0.0	x = 186.3 cm	x = 170.0 cm	16.3
Sum			89.6			201.9

Table 5. Comparison of the Difference Between the Position of the Coupled and Uncoupled 50% Isochlors for Both the Modified and Standard Henry Saltwater Intrusion Problem

	Standard			Modified		
	Coupled	Uncoupled	Difference (δ)	Coupled	Uncoupled	Difference (δ)
z = 0 cm	x = 138.0 cm	x = 149.4 cm	11.4	x = 107.3 cm	x = 128.7 cm	21.4
z = 10 cm	x = 140.8 cm	x = 150.1 cm	9.3	x = 109.9 cm	x = 129.1 cm	19.2
z = 20 cm	x = 147.0 cm	x = 151.7 cm	4.7	x = 116.7 cm	x = 130.6 cm	13.9
z = 30 cm	x = 155.2 cm	x = 154.6 cm	0.6	x = 126.8 cm	x = 133.5 cm	6.7
z = 40 cm	x = 165.0 cm	x = 158.8 cm	6.2	x = 139.0 cm	x = 137.6 cm	1.4
z = 50 cm	x = 175.0 cm	x = 164.6 cm	10.4	x = 152.4 cm	x = 143.2 cm	9.2
z = 60 cm	x = 184.1 cm	x = 171.9 cm	12.2	x = 166.0 cm	x = 150.5 cm	15.5
z = 70 cm	x = 190.7 cm	x = 181.3 cm	9.4	x = 178.5 cm	x = 160.0 cm	18.5
z = 80 cm	x = 195.4 cm	x = 194.0 cm	1.4	x = 188.8 cm	x = 172.4 cm	16.4
z = 90 cm	x = 197.1 cm	x = 197.1 cm	0.0	x = 195.2 cm	x = 188.1 cm	7.1
z = 100 cm	x = 197.3 cm	x = 197.5 cm	0.2	x = 195.9 cm	x = 196.1 cm	0.2
Sum			65.8			129.5

numerator, to the density-driven flux in the denominator. Under conditions when $a \sim 1$ the fluxes are balanced, when $a \gg 1$ the system is advective dominant and where $a \ll 1$ then the system is dominated by density-driven processes. For the simulations presented in this work, $a = 0.263$ (standard) and $a = 0.1315$ (modified) thereby suggesting that both systems are weakly density-dependent with the modified system being more so density-dependent than the standard version. Therefore it is physically justifiable that a reduction in recharge corresponds to an increase in the relative importance of density-dependent characteristics of the Henry problem. This kind of physical reasoning agrees with the intuitive reasoning outlined in section 5.3 where it was claimed that the worthiness of the Henry problem could be increased by either decreasing the recharge Q, or increasing the density of the invading fluid ρ_s.

5.6. Supplementary Model Testing Data

[36] To supplement the presentation of the numerical and semianalytical isochlors for the modified Henry problem, it has been suggested that the velocity field should also be computed and visualized in the benchmarking process as a qualitative check on the internal mixing environment [*Simpson and Clement*, 2003]. The fully-coupled steady state velocity field with the 25%, 50% and 75% isochlors superimposed are presented in Figure 3. As expected, the velocity field shows the saline fluid invading deep in the aquifer and recirculating to form a single overturning cell. The position and direction of the velocity vectors where the fluid stagnates and recirculates corresponds well with the shape of the coupled isochlors. Additionally the velocities near the outflow region are relatively large, as expected, since this is a region of convergent flow.

[37] Further to the presentation of the steady state isochlors and velocity field, it is possible to use transient data to aid in the benchmarking of a numerical algorithm [*Simpson and Clement*, 2003]. This is an important point to make since one of the disadvantages of the Henry problem is that the solution deals only with a steady state profile. In reality, most density-dependent groundwater models would be required to be used in both steady state and transient situations. Following the work of *Frind* [1982], *Simpson and Clement* [2003] suggested that it is also possible to use the results from a transient model in conjunction with the Henry problem to provide data to conduct an intercode comparison. The temporal advancement of the 25%, 50% and 75% isochlors along the base of the aquifer for the modified problem is presented in Figure 4. It should be stated that it is only possible to use this data to conduct an intercode comparison if the

Table 6. Comparison of the Difference Between the Position of the Coupled and Uncoupled 75% Isochlors for Both the Modified and Standard Henry Saltwater Intrusion Problem

	Standard			Modified		
	Coupled	Uncoupled	Difference (δ)	Coupled	Uncoupled	Difference (δ)
z = 0 cm	x = 159.0 cm	x = 160.5 cm	1.5	x = 140.0 cm	x = 143.7 cm	3.7
z = 10 cm	x = 161.8 cm	x = 161.3 cm	0.5	x = 143.0 cm	x = 144.5 cm	1.5
z = 20 cm	x = 168.2 cm	x = 163.5 cm	4.7	x = 150.1 cm	x = 146.3 cm	3.8
z = 30 cm	x = 176.4 cm	x = 166.8 cm	9.6	x = 160.1 cm	x = 149.4 cm	10.7
z = 40 cm	x = 183.9 cm	x = 171.5 cm	12.4	x = 170.0 cm	x = 153.8 cm	16.2
z = 50 cm	x = 190.0 cm	x = 177.3 cm	12.7	x = 179.9 cm	x = 160.0 cm	19.9
z = 60 cm	x = 193.8 cm	x = 185.1 cm	8.7	x = 187.7 cm	x = 167.9 cm	19.8
z = 70 cm	x = 196.6 cm	x = 194.5 cm	2.1	x = 192.8 cm	x = 178.3 cm	14.5
z = 80 cm	x = 197.5 cm	x = 197.3 cm	0.2	x = 196.4 cm	x = 191.4 cm	5.0
z = 90 cm	x = 198.2 cm	x = 198.5 cm	0.3	x = 197.5 cm	x = 197.5 cm	0.0
z = 100 cm	x = 198.7 cm	x = 198.5 cm	0.2	x = 198.0 cm	x = 198.2 cm	0.2
Sum			52.9			95.3

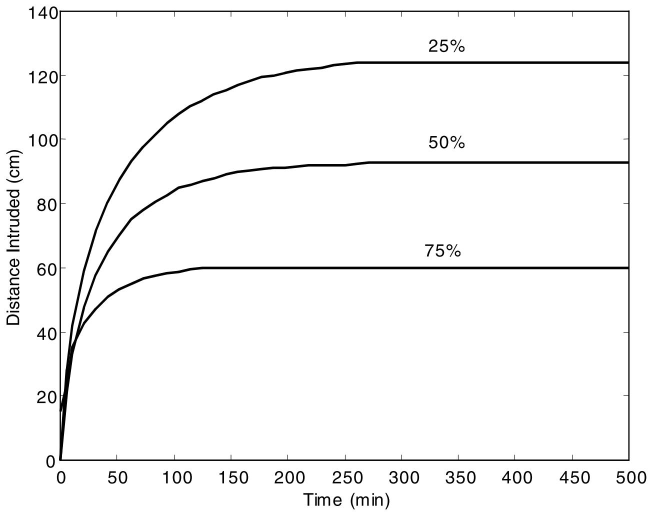

Figure 4. Transient position of the intersection of the 25%, 50%, and 75% isochlors with the base of the aquifer for the modified Henry saltwater intrusion problem a = 0.1315, b = 0.2, and ξ = 2.0.

problem is executed using the same initial conditions as was done in this study.

6. Discussion and Conclusions

[38] In comparing the coupled and uncoupled solutions to the standard and modified Henry problems, it was demonstrated that the difference between the coupled and uncoupled profiles was more pronounced for the modified case. This difference indicates that the relative importance of the density effects is greater for the modified case where the influence of the boundary forcing is reduced. Therefore it is possible to invest the same amount of effort required to simulate the standard Henry problem and obtain the modified solution where the density effects are more pronounced. This increase in worthiness is simply achieved by reducing the freshwater recharge. The modified solution retains all the advantages of the usual Henry problem with the additional benefits of having an improved worthiness according to the coupled versus uncoupled criteria. Until now, there has been no investigation into how to overcome the difficulties associated with the limited worthiness of the standard Henry problem. Therefore this improvement represents a major advancement in the evolution of the Henry saltwater intrusion problem as a more rigorous test case of density-dependent groundwater flow models.

[39] The modified problem also has the advantage of being able to be benchmarked against the semianalytical solution presented here. Using a semianalytical solution to test the results of a numerical code is preferred over using intercode comparisons since intercode comparisons will not necessarily identify all sources of possible error. For example, the solution of *Frind* [1982] has been used to check the results of several published codes [e.g., *Huyakorn et al.*, 1987; *Galeati et al.*, 1992; *Boufadel et al.*, 1999]. Many of these codes have demonstrated that identical results can be obtained when using similar numerical discretizations. The work of *Croucher and O'Sullivan* [1995] showed that the results of *Frind* [1982] might be subject to numerical dispersion errors since a grid refinement procedure showed that the results obtained using a refined grid were significantly different to the results of *Frind* [1982]. Therefore all of the codes that were deemed correct according to the results of *Frind* [1982] are also presumably subject to the same uncertainty. This problem of model testing via intercode comparisons can be avoided through the use of a semianalytical method as proposed in this work because the semianalytical results are not subject to numerical errors.

[40] It should be mentioned that certain limitations of the Henry problem are not addressed in this study. For example, this is a theoretical investigation and does not involve any physical data analysis that has become popular with other density-dependent benchmark standards [e.g., *Simmons et al.*, 1999; *Johannsen et al.*, 2002]. Additionally, the Henry problem is also limited because there is no inclusion of heterogeneity which can be important under certain density-dependent flow conditions [*Simmons et al.*, 2001].

[41] To summarize, this work has shown that in general it is possible to use the proposed modified Henry problem with no more effort than is required for the simulation of the standard Henry problem. The modified problem retains the prior benefits of the standard case; however, there is a significant benefit in the worthiness of the proposed test case and also an increase in the ease with which the numerical results for the modified case can be tested using the semianalytical results.

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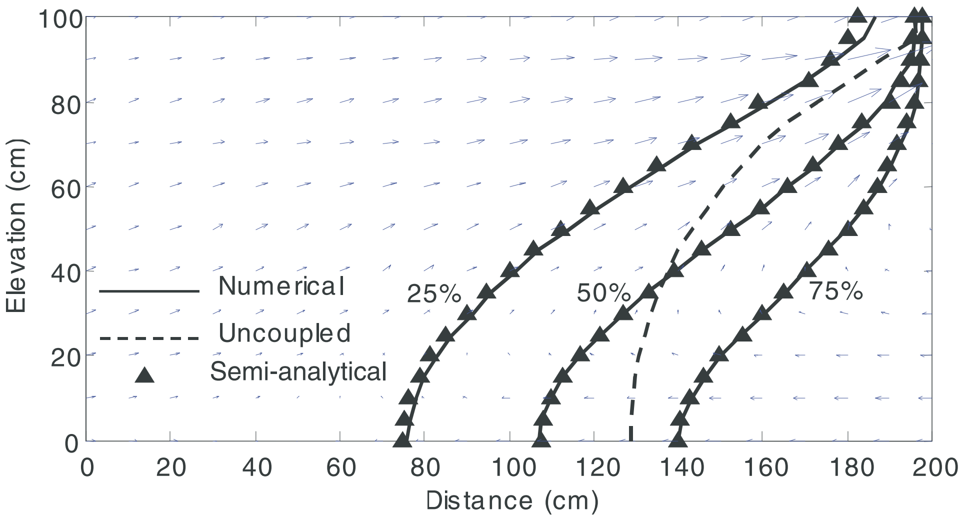

Figure 3. Comparison of numerical and semianalytical results for the modified Henry saltwater intrusion problem a = 0.1315, b = 0.2, and ξ = 2.0. The velocity distribution and uncoupled 50% numerical isochlor (dashed) are also shown.